

Workshop on Complex Geometry in Osaka 2024

-Hodge theory and vanishing theorem-

Date: 13th–15th March 2024. (2024年3月13–15日)

Place: Lecture Room E404 in Graduate School of Science Building E in Osaka University (Toyonaka Campus). (大阪大学理学部 E404 講義室 (豊中キャンパス))

Program

3/13 (Wednesday)

13:30-14:30 Shouhei Ma (Tokyo Institute of Technology)

Mixed Hodge structures of locally symmetric varieties

14:45-15:45 Yongpan Zou (The University of Tokyo)

On the Kodaira-Saito Vanishing Theorem for Weakly Ample Divisors

16:15-17:15 Yuta Watanabe (The University of Tokyo)

Nakano-Nadel type, Bogomolov-Sommese type vanishing involving multiplier ideals

3/14 (Thursday)

10:00-11:00 Osamu Fujino (Kyoto University)

Vanishing theorems for projective morphisms between complex analytic spaces

11:15-12:15 Yota Shamoto (Waseda University)

Stokes structure of difference modules

14:15-15:15 Takashi Ono (Osaka University)

Wild harmonic bundles with skew-symmetric structure

15:45-16:45 Shin-ichi Matsumura (Tohoku University)

An injectivity theorem on snc compact Kahler spaces: an application of the theory of harmonic integrals on log-canonical centers via adjoint ideal sheaves

3/15 (Friday)

10:00-11:00 Sheng Rao (Wuhan University)

Geometry of logarithmic forms and deformations of complex structures

11:15-12:15 Takahiro Saito (Chuo University)

mixed Hodge modules of normal crossing type on a smooth toric variety

Organizers

- Masataka Iwai (Osaka University)
- Hisashi Kasuya (Osaka University)

Supports

- JSPS KAKENHI 22K13907 Grant-in-Aid for Early Career Scientists.
- JSPS KAKENHI 19H01787 Grant-in-Aid for Scientific Research (B)

Homepage

We have posted various information on our website, including how to access to the conference room "Lecture Room E404".

Homepage Link: https://masataka123.github.io/complexgeometry_osaka_2024/

You can also read the QR code below:



Abstract

3/13 (Wednesday)

Shouhei Ma (Tokyo Institute of Technology)

Mixed Hodge structures of locally symmetric varieties

I will talk about the mixed Hodge structures on the cohomology of locally symmetric varieties. In the middle degree, I relate the weight filtration to the Siegel operators for certain modular forms. This has an application to a classical problem on the Siegel operators. In the general degrees, I construct a spectral sequence which converges to the edge Hodge components in the Hodge triangle, and whose E1 page is expressed by some simple geometric invariants associated to the cusps. This already degenerates at E1 in a certain range. Applications contain a new proof of Harder on the Eisenstein cohomology of Hilbert modular varieties.

Yongpan Zou (The University of Tokyo)

On the Kodaira-Saito Vanishing Theorem for Weakly Ample Divisors.

On a smooth projective variety, considering a reduced effective divisor that is weakly ample in the sense of cohomology, we introduce a Kodaira vanishing theorem for it. Our approach involves utilizing the Hodge ideal introduced by Mustata-Popa.

Yuta Watanabe (The University of Tokyo)

Nakano-Nadel type, Bogomolov-Sommese type vanishing involving multiplier ideals

In this talk, we first obtain the Bogomolov-Sommese type vanishing theorem involving multiplier ideal sheaves for big line bundles. We investigate Griffiths and (dual) Nakano positivity for singular Hermitian metrics of holomorphic vector bundles and obtain various vanishing theorems.

3/14 (Thursday)

Osamu Fujino (Kyoto University)

Vanishing theorems for projective morphisms between complex analytic spaces

We discuss vanishing theorems for projective morphisms between complex analytic spaces and some related results. They will play a crucial role in the minimal model theory for projective morphisms of complex analytic spaces. Roughly speaking, we establish an ultimate generalization of Kollár's package from the minimal model theoretic viewpoint.

Yota Shamoto (Waseda University)

Stokes structure of difference modules

P. Deligne and B. Malgrange introduced the notion of Stokes filtered local systems as the Betti counterpart of meromorphic connection stalks at a point with irregular singularity in one variable. This talk introduces an analogous notion, i.e., the Betti counterpart for additive difference modules. We also explain some motivation from non-abelian Hodge theory for periodic monopoles.

Takashi Ono (Osaka University)

Wild harmonic bundles with skew-symmetric structure

There is a one-on-one correspondence between a good wild harmonic bundle and a polystable good filtered Higgs bundle with vanishing Chern classes, a branch of Kobayashi-Hitchin correspondence. In this talk, I want to show how the Higgs bundle decomposes when the harmonic bundle has skew-symmetric pairing. Also, I want to show the Kobayashi-Hitchin correspondence with skew-symmetry.

Shin-ichi Matsumura (Tohoku University)

An injectivity theorem on snc compact Kahler spaces: an application of the theory of harmonic integrals on log-canonical centers via adjoint ideal sheaves

In this talk, I would like to discuss the injectivity theorem, an extension of the Kodaira vanishing theorem into the 'semi-positive' case from a complex analytic perspective. Initially, I will review Kollar's formulation of injectivity for semi-ample line bundles in algebraic geometry and its generalization to semi-positive line bundles by Enoki in the complex analytic setting. Subsequently, my focus will shift to the injectivity theorem for log canonical (LC) pairs, established by Ambro and Fujino in the context of algebraic geometry using Hodge theory. I would like to explain our proof for Fujino's conjecture, which asks for a complex analytic analog of Ambro and Fujino's result. This is joint work with Tsz On Mario Chan and Young-Jun Choi (Pusan National University).

3/15 (Friday)

Sheng Rao (Wuhan University)

Geometry of logarithmic forms and deformations of complex structures

We present a new method to solve certain dbar-equations for logarithmic differential forms by using harmonic integral theory for currents on Kahler manifolds. As applications, we generalize the result of Deligne about closedness of logarithmic forms, give geometric and simpler proofs of Deligne’s degeneracy theorem for the logarithmic Hodge to de Rham spectral sequences at E1-level, as well as certain injectivity theorem on compact Kahler manifolds. Our method also plays an important role in Cao–Paun’s recent works on the extension of pluricanonical sections and proof of Fujino’s injectivity conjecture. Furthermore, for a family of logarithmic deformations of complex structures on Kahler manifolds, we construct the extension for any logarithmic (n,q) -form on the central fiber and thus deduce the local stability of log Calabi–Yau structure by extending an iteration method to the logarithmic forms. Finally we prove the unobstructedness of the deformations of a log Calabi–Yau pair and a pair on a Calabi–Yau manifold by differential geometric method. Its projective case was originally obtained by Katzarkov–Kontsevich–Pantev in 2008.

Takahiro Saito (Chuo University)

mixed Hodge modules of normal crossing type on a smooth toric variety

In general, mixed Hodge modules (MHMs) are complicated and difficult to deal with, while the general theory is well established. However, if the underlying D-module of a MHM on C^n satisfies the condition: “of normal crossing type”, it can be expressed in a linear algebraic way. This is a generalization to MHM of the well-known “can-var description” of perverse sheaves on C . As an application, we can give a natural definition of “the Fourier-Laplace transform of a MHM of normal crossing type”.

In the first half, I will introduce these facts. In the second half, I will talk on the recent progress on the generalization of them to MHMs on smooth toric varieties and the application to the “Hodge structure” of the Fukaya category of the plumbed manifolds of a Dynkin diagram (in collaboration with Tatsuki Kuwagaki).