6th Tropical Geometry Workshop

Workshop name (JP): 第6回トロピカル幾何ワークショップ
Date: March 14th - 15th, 2024
Venue: Hiroshima University, Higashi-Hiroshima campus, School of Science,
Room E104 (広島大学 東広島キャンパス 理学部棟 E104)
https://www.hiroshima-u.ac.jp/en/access/higashihiroshima/busstop_higashihiroshima/aca_1
Presentation Method: Chalkboard, slides for in-person presentation
Speakers: Hajime Fujita (Japan Women's University)
Keita Goto (National Taiwan University)
Paul Alexander Helminck (Tsukuba University)
Fumihiko Sanda (Gakushuin University)
JuAe Song (Kyoto University)
Masayuki Sukenaga (Hiroshima University)
Yuki Tsutsui (University of Tokyo)
Yuto Yamamoto (RIKEN iTHEMS)
Takahiko Yoshida (Meiji Unviersity)
Organizers: Masayuki Sukenaga (Hiroshima University)
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Program

March 14th (Thu)

9:30 - 10:30 Yuki Tsutsui
Title: The Riemann–Roch number of tropical curves in moderate position on tropical surfaces.
Abstract:
Thanks to the theory of Chern classes for tropical manifolds by de Medrano–Rincón–Shaw, it has become possible to define the Todd classes for tropical manifolds. Particularly, for a divisor on a compact tropical manifold, the Riemann–Roch number can now be defined as the product of its tropical first Chern class and the Todd class. Classically, for divisors on projective complex manifolds, this number corresponds to the Euler characteristic of the cohomology of the associated line bundle by the Hirzebruch–Riemann–Roch theorem. A tropical analogue of the Riemann–Roch theorem for divisors on compact tropical curves was shown by Gathmann–Kerber and Mikhalkin–Zharkov as a generalization of the Riemann–Roch theorem for finite graphs by Baker–Norine, but a higher-dimensional version of their results has not yet been formulated. Therefore, the geometric meaning of the Riemann–Roch number of divisors on tropical manifolds is still not well understood.

In this talk, we discuss a conjecture that, if a divisor on on a compact tropical manifold is a tropical submanifold in the sense defined by de Medrano–Rincón–Shaw and in moderate position, then its Riemann–Roch number has a geometric explicit formula. I will also explain that this conjecture is indeed true for compact tropical surfaces admitting Delzant face structures.

10:45 - 11:45 Hajime Fujita

Title: A "generalized" generalized Pythagorean theorem on a compactification of the dually flat manifold via toric geometry
Abstract:

A dually flat manifold is a Riemannian manifold equipped with two flat connections which are dual to each other. It was introduced by Amari-Nagaoka in information geometry to describe a geometric structure of a family of probability densities such as an exponential family. Any dually flat manifold associates a function called the Bregman divergence. The divergence can be seen as a generalization of the square of the distance function on a Riemannian manifold. In fact, there exists a formula called the generalized Pythagorean theorem, which has applications in statistical inference.

The interior part of any convex polytope has natural dually flat structure. In this talk we first give the description of the dually flat structure and explain the extension of the dually flat structure to the boundary. After that we focus on Delzant polytopes as a special case, which is given by the moment map image of a toric Kahler manifold.

We give a generalization of the generalized Pythagorean theorem including the boundary points on the Delzant polytope. This talk is based on the paper arXiv:2305.08422 [math.SG] (published in Information Geometry, 2023).

13:20 - 14:20 Takahiko Yoshida
Title: Geometric quantization of Lagrangian torus fibrations and adiabatic limit
Abstract:

In a series of studies on localization of the index of a Dirac-type operator on a torus fibration [1, 2, 3], Fujita-Furuta-Yoshida showed that the index of a Spin^c Dirac operator on a compact Lagrangian torus fibration with prequantization line bundle is localized on Bohr-Sommerfeld fibers and singular fibers.

In this talk, when a not necessarily compact, non-singular Lagrangian torus fibration with complete base admits a compatible complex structure, we explain that, under technical conditions on the complex structure, we can concretely construct a complete orthogonal system $\{\vartheta_b\}_{b:\ \text{BS-point}}$ of the space of holomorphic L^2-sections of the prequantization line bundle subscripted by the Bohr-Sommerfeld points such that each ϑ_b converges to the delta-function section supported on the Bohr-Sommerfeld fiber at b under adiabatic limit. In the case of an abelian variety, ϑ_b coincides with Jacobi's Theta function up to constant. If time permits, we would also like to explain recent developments, such as the irreducible decomposition of the representation of the automorphic group on the space of holomorphic L^2-sections.

Part of this talk is based on [4].

References

[1] H. Fujita, M. Furuta, and T. Yoshida, *Torus fibrations and localization of index I*, J. Math. Sci. Univ. Tokyo **17** (2010), no. 1, 1-26.
[2] ———, *Torus fibrations and localization of index II*, Comm. Math. Phys. **326** (2014), no. 3, 585-633.
[3] ———, *Torus fibrations and localization of index III*, Comm. Math. Phys. **327** (2014), no. 3, 665-689.
[4] T. Yoshida, *Adiabatic limits, theta functions, and geometric quantization*, arXiv:1904.04076, 2019.

14:40 - 15:40 Keita Goto
Title: A non-Archimedean approach to get Hessian manifolds
Abstract:

Mirror symmetry, which was originally predicted in theoretical physics, is nowadays a major open problem in algebraic geometry. In this context, people construct Hessian manifolds (with singularities) for polarized Calabi-Yau manifolds to get what are referred to as mirror pairs. Once given a degenerating family of polarized Calabi-Yau varieties,

one can consider a limit of such Hessian manifolds. This approach does highly depend on differential geometry. In order to understand this in terms of algebraic geometry, one may adopt a non-Archimedean approach. In this talk, I will explain this approach and compare it with the original approach for polarized abelian varieties. This talk is based on a joint work with Yuji Odaka.

16:00 - 17:00 Yuto Yamamoto
Title: Non-archimedean SYZ fibrations via tropical contractions
Abstract:

For a maximally degenerate Calabi–Yau variety, the Berkovich retraction associated with a (good) minimal dlt model is regarded as an SYZ fibration in non-archimedean geometry. In general, the integral affine structure induced on the base space of the fibration differs from the one defined for the dual intersection complex of a toric degeneration in the Gross–Siebert program. In this talk, using tropical geometry, we construct non-archimedean SYZ fibrations whose bases are integral affine manifolds appearing in the Gross–Siebert program for Calabi–Yau complete intersections of Batyrev–Borisov.

March 15th (Fri)

10:20 - 11:20 Fumihiko Sanda
Title: Tropical geometry and equivariant Lagrangian Floer cohomology
Abstract:

Let X_P be a toric manifold defined by a Delzant polytope P, and $\Lambda\langle P\rangle$ be the affinoid algebra corresponding to P (the ring of convergent power series on P). Using Lagrangian Floer theory, we can define a function $W \in \Lambda\langle P\rangle$ which is called a potential function. For a Lagrangian torus orbit L corresponding to the tropicalization of critical points of W, Floer cohmology $HF(L)$ does not vanish and L is Hamiltonian non-displaceable. In this talk, I will explain an equivariant version of this theory. This is a joint work with Masahiro Futaki.

13:20 - 14:20 Masayuki Sukenaga
Title: Smooth tropical complete intersection curves of genus 3 in $\mathbb{R}^3$
Abstract:

We develop a method for describing the tropical complete intersection of a tropical hypersurface and a tropical plane in $\mathbb{R}^3$. This involves a method for determining the topological type of a complete intersection by using a polyhedral complex. As an application, we show that there are no smooth tropical complete intersection curves in $\mathbb{R}^3$ whose skeletons are the lollipop graph of genus 3. This gives a partial answer to the open question of whether a lollipop graph of genus 3 appears in the skeleton of a tropical curve on a tropical plane in $\mathbb{R}^3$.

14:40 - 15:40 JuAe Song
Title: A characterization of rational function semifields of tropical curves
Abstract:

In the classical algebraic geometry, it is well-known that function fields of algebraic curves are characterized with transcendental degree. On the other hand, since there is no appropriate analogue of transcendental degree for semifield extensions, we cannot characterize rational function semifields of (abstract) tropical curves as in the classical case. In this talk, we give a characterization of rational function semifields of tropical

curves with geometry over the tropical rational function semifields. If time allows, we also give the strategy of its proof.

16:00 - 17:00 Paul Alexander Helminck
Title: Toric ranks and component groups of Jacobians of modular curves
Abstract:

Let p be a prime number not equal to 2 or 3, and let H be a congruence subgroup in $SL_2(\mathbb{Z})$ with modular curve X_H/K. In this talk, I will show how to explicitly reconstruct a deformation retract of the Berkovich analytification of X_H at a finite place of K over p in terms of glued Hecke double coset spaces. This will allow us to give the semistable toric rank and component group of the Jacobian of X_H, as well as its non-abelian l-Selmer groups for l coprime to p.

In the talk, I will explicitly determine the skeleta of the modular curves $X_0(N)$. For $N = p^n$ and $n \leq 4$, this in particular recovers results by Deligne–Rapoport, Edixhoven, Coleman–McMurdy and Tsushima. Using these descriptions, I will show how to obtain the prime-to-6 parts of the component groups of $X_0(N)$ over the field extension given by Krir's theorem. This generalizes results by Mazur and Rapoport from the squarefree case to general N.