Holomorphic Floer theory and related topics, 2023[1]

Date: 24 October – 27 October 2023

Venue: Kyoto University, Room 127 in Graduate School of Science Bldg. No.3 = $\boxed{5}$ in the campus map: https://www.kyoto-u.ac.jp/en/access/north-campus-map

Organizers: Tatsuki Kuwagaki (Kyoto), Hiroshi Ohta (Nagoya)

Program: The time schedule on 26 is irregular, because the room is used for other purpose around noon.

	24 Oct.	25 Oct.	26 Oct.	27 Oct.
10:00-11:00		Gammage 2	Kuwagaki(10:30-11:30)	Rezchikov 3
11:30-12:30		Nho 2		Shibata 2
13:00-14:00	Gammage 1		Rezchikov 2(13:15-14:15)	
14:30-15:30	Nho 1	Gammage 3	Shibata 1(14:45-15:45)	
16:00-17:00	Rezchikov 1	Kinjo 1	Kinjo 2(16:15-17:15)	

Benjamin Gammage (Harvard)

1,2,3. *Perverse schobers and hypertoric varieties*

Predictions from physics suggest that we should be able to assign a 2-category to a holomorphic symplectic manifold (possibly gauged by a Hamiltonian group action). Justin Hilburn, Aaron Mazel-Gee, and I suggested that the perverse schobers of Kapranov-Schectman furnish such a 2-category. In the first talk, we will introduce this 2-category, along with some known and expected structural properties. In the following talks, we will study the example of hypertoric varieties, which play the same role in holomorphic symplectic geometry that toric varieties play in symplectic geometry: namely, they provide an extensive class of examples whose geometry can be described in simple combinatorial terms. We will give a complete, explicit calculation of the 2-category of perverse schobers on hypertoric varieties. This gives a conjectural prediction for the Fueter 2-category of a hypertoric variety, including T^*P^n or the resolution of an A_n surface singularity.

Yoon Jae Nho (Cambridge)

1. *Spectral networks for meromorphic quadratic differentials*

   We will investigate the spectral network structure for meromorphic quadratic differentials and describe the GMN non-abelianization map.
2. *Non-abelianization and family Floer cohomology local system*

   We will study how to relate the GMN non-abelianization map to the family Floer cohomology local system of the rank 2 spectral curve.

Semon Rezchikov (Princeton)

1. *Complex Morse Theory*

   A real Morse-Smale function on a Riemannian manifold defines an associated algebraic invariant: a homology group. Complexifying this setup, one finds that a holomorphic Morse function W on a Kahler manifold should be associated to a richer algebraic invariant, namely, a category. This category should categorify the real Morse homologies of the real parts $\mathrm{Re}(e^{i\theta}W)$, and should give a definition of the Fukaya-Seidel category of W, thought of as a symplectic Lefshetz fibration, without reference to Lagrangian submanifolds. We will discuss what is known and what are open problems regarding this construction.

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2. *Categorical Aspects of the Fueter Equation*
   A Kahler structure on a manifold defines Morse functions and metrics on the spaces of paths between Lagrangian submanifolds of this manifold. The algebraic invariants of this Morse function are known to assemble into a richer algebraic invariant: the Fukaya category. Complexifying this setup, a hyperkahler structure on a manifold defines complex Morse functions and Kahler metrics on path spaces between holomorphic Lagrangians in this manifold, which should define a very rich algebraic invariant: a 2-category. We will describe this conjectural picture, and discuss its consequences, some of which can be proven.
3. *The Fueter 2-category: examples and connections*
   In this talk, we will discuss further aspects of the Fueter 2-category, including interesting analytical problems and connections to other areas of mathematics, which are of analytic, geometric, and algebraic nature.

Tasuki Kinjo (RIMS, Kyoto)

1. *An introduction to Donaldson–Thomas perverse sheaves*
   In the first talk, I will provide a brief introduction to the theory of Donaldson—Thomas perverse sheaves introduced by Joyce and his collaborators, which can be seen as a version of holomorphic analogue of Floer theory. The primary goal of this talk is to explain Joyce's conjecture concerning the functorial behavior of the Donaldson—Thomas perverse sheaves.
2. *Derived microlocal geometry and its applications*
   In the second talk, I will discuss a joint work with Adeel Khan on a derived algebro-geometric generalization of microlocal sheaf theory. We will discuss a relationship between this formalism and Donaldson—Thomas perverse sheaves and explain how this can be used to solve a special case of Joyce's conjecture explained in the first talk.

Tatsuki Kuwagaki (Kyoto)

1. *Exact WKB analysis, sheaf, and Floer theory*
   In this talk, I would like to explain the relation between exact WKB analysis and Floer theory from the perspective of the theory of sheaf quantizations. This talk is partly based on an ongoing joint work with Hiroshi Ohta and Kohei Iwaki.

Taisuke Shibata (RIMS, Kyoto)

1. *An introduction to embedded contact homology*
   In the 1990s, Taubes showed the equivalence between the number of certain pseudo-holomorphic curves (Gromov invariants) and Seiberg-Witten invariants on symplectic 4-manifolds. Embedded Contact Homology (ECH) introduced by M. Hutchingsis a three-dimensional Floer homology analog of Gromov invariants and is defined for contact 3-manifolds. ECH has become a powerful tool in the study of three-dimensional Reeb flows. It is well-known to be isomorphic to Monopole Floer homology defined by Kronheimer and Mrowka. In this talk, I will explain the construction and basic properties of ECH.
2. *Disk-like Birkhoff sections and the ECH spectrum on lens spaces*
   Prior to the introduction of ECH, important research was conducted by Hofer, Wysocki, and Zehnder. They constructed disk-like Birkhoff sections on convex three-dimensional contact spheres from pseudo-holomorphic curves. Recently, it was generalized to certain lens spaces by developing the original technique. In this talk, I will explain how the results of Wysocki, and Zehnder are related to ECH. Specifically, I will introduce the approach using ECH to the existence of disk-like Birkhoff sections in lens spaces. Additionally, I will also discuss the connection with the ECH spectrum.