力学系勉強会

下記の要領で力学系勉強会を開催します。今回は京都大学の塚本真輝さん、埼玉大学の高橋悠樹さん、パリ大学の Biebler さんが講師です。時間をかけて基礎的な部分から講義していただく予定なので、学生の方の積極的な参加をお待ちしています。また、旅費の補助を希望される方は、1月27日(金)までに石井にご連絡下さい。

日時: 3月20日(月)10:00から24日(金)17:00まで

会場: 九州大学伊都キャンパス理学部、C棟5階 C-501 講義室(月曜のみ)、

D棟4階 D-413 オーディトリアム (火曜から金曜)

プログラム:

3月20日(月)

10:00-11:30 塚本真輝: Variational principle in mean dimension theory, I.

13:30-15:00 高橋悠樹: Lyapunov exponent for SL(2, ℝ) cocycles, I.

15:30-17:00 Sebastien Biebler: Emergence of wandering stable components, I.

3月21日(火)

10:00-11:30 塚本真輝: Variational principle in mean dimension theory, II.

13:30-15:00 高橋悠樹: Lyapunov exponent for SL(2, ℝ) cocycles, II.

15:30-17:00 Sebastien Biebler: Emergence of wandering stable components, II.

3月22日(水)

10:00-11:30 塚本真輝: Variational principle in mean dimension theory, III.

13:30-15:00 高橋悠樹: Lyapunov exponent for SL(2, ℝ) cocycles, III.

15:30-17:00 Sebastien Biebler: Emergence of wandering stable components, III.

3月23日(木)

10:00-11:30 塚本真輝: Variational principle in mean dimension theory, IV.

13:30-15:00 高橋悠樹: Lyapunov exponent for SL(2, ℝ) cocycles, IV.

15:30-17:00 Sebastien Biebler: Emergence of wandering stable components, IV.

3月24日(金)

10:00-11:30 塚本真輝: Variational principle in mean dimension theory, V.

13:30-15:00 高橋悠樹: Lyapunov exponent for $SL(2,\mathbb{R})$ cocycles, V.

15:30-17:00 Sebastien Biebler: Emergence of wandering stable components, V.

ただしこのプログラムは講義の進行状況などに応じて適宜変更される可能性 があります。

講義内容:

塚本真輝 (京都大学)

タイトル: Variational principle in mean dimension theory.

アブストラクト: "Mean dimension" is a topological invariant of dynamical systems introduced by Gromov in 1999. It counts the number of parameters per unit time for describing the given dynamical system. "Variational principle" is a framework for connecting topological dynamics and ergodic theory. The most famous variational principle is a one in entropy theory: Topological entropy is equal to the supremum of the Kolmogorov-Sinai entropy over all invariant probability measures. The purpose of my talk is to explain a variational principle in mean dimension theory. It turns out that rate-distortion function, a fundamental limit of lossy data compression, plays a crucial role. The plan of my talk is as follows:

- (1) The survey of some highlights of mean dimension theory.
- (2) Basics of rate distortion theory.
- (3) Variational principle and the idea of the proof.

高橋悠樹 (埼玉大学)

タイトル: Lyapunov exponent for $SL(2,\mathbb{R})$ cocycles.

 $\mathcal{T}\mathcal{T}\mathcal{A}\mathcal{h}\mathcal{D}\mathcal{h}$: We will define the Lyapunov exponent for $\mathrm{SL}(2,\mathbb{R})$ cocycles, and discuss its basic properties. If time allows, we will discuss the speaker's recent result about the invariant measures for iterated function systems with inverses.

Sebastien Biebler (Université Paris Cité)

タイトル: Emergence of wandering stable components.

アブストラクト:

I. Newhouse phenomenon

In the 60s, in a mathematical optimistic movement aiming to describe a typical dynamical system, Smale conjectured the density of uniform hyperbolicity in the space of C^r -diffeomorphisms f of a compact manifold M. In 1974, Newhouse discovered an extremely complicated new phenomenon, resulting in an obstruction to Smale's conjecture. Specifically, Newhouse showed that for every $2 \le r \le \infty$, there is an open set U in the space of C^r -diffeomorphisms of M such that a generic map f in U has infinitely many attracting periodic points. In particular, the dynamics of such systems is very difficult to describe.

In this talk, I will define precisely the Newhouse phenomenon and explain how it works.

II. Newhouse phenomenon in the complex setting

In this talk, I will discuss some generalizations of Newhouse's result to the complex setting.

III. and IV. Wandering Fatou components for Hénon maps (Parts 1 and 2)

A celebrated theorem of Sullivan in the 80s shows that any rational map of the Riemann sphere does not have a wandering Fatou component. In particular, every Fatou component is preperiodic. Since the possible dynamics on periodic Fatou components are classified, this allows to give a complete description of the dynamics on the Fatou set.

In a recent joint work with Pierre Berger, we show that there exist polynomial automorphisms of \mathbb{C}^2 (more precisely Hénon maps) displaying a wandering Fatou component. We also study the statistical behavior of orbits of points inside the wandering component, and we show that it is very difficult to describe. In these two talks, I will discuss the main ideas of the proof of this result, based mainly on the Newhouse phenomenon.

V. Bifurcations in two complex variables

In the 80s Lyubich and Mané–Sad–Sullivan showed that for a family of rational maps, the continuity of the Julia set in the Hausdorff topology with the parameter is equivalent to the holomorphic motion of the repelling periodic points. This classical result allows to split the parameter set into a stability locus and a bifurcation locus. An important feature is that the stability locus is a dense open subset of the parameter set.

In higher dimension, this description has been extended by Berteloot–Bianchi–Dupont and Dujardin–Lyubich. Contrarily to the one-dimensional case, the bifurcation locus can have a non empty interior: it is possible to find robust bifurcations.

In this talk, I will discuss these results and robust bifurcations in the particular case of Lattès maps.

I is an introduction to Newhouse phenomenon, which is essential for the proof of the wandering component for the Hénon map. In II I can discuss in particular the gap lemma. III and IV focus on the proof of the wandering component for the Hénon map and in V I can talk about the bifurcations of Lattès maps among other things.

本研究集会は、九州大学マス・フォア・イノベーション卓越大学院プログラムと科学研究費基盤研究 (B) (20H01809) の補助の元に開催します。

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