Closed four dimensional Ricci flow with integral bounds of the scalar curvature

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In this talk, we will discuss extension problem on the Ricci flow with bounded some sort of geometric quantity. In 1982, Hamilton introduced the Ricci flow which deforms Riemannian metrics in the direction of the Ricci tensor. One hopes that the Ricci flow will deform any Riemannian metric to some canonical metrics. Indeed, Hamilton proved that any Riemannian metric on closed three manifold with positive Ricci curvature can be deformed (by the normalized Ricci flow) into a metric with positive constant sectional curvature. He also proved that the Ricci flow equaiton on a closed manifold has a unique short time solution for any initial metric. The next immediate question is the so-called "maximal existence time" for the Ricci flow with respect to initial metric. Hamilton proved that $T < +\infty$ is the maximal existence time of a closed Ricci flow $(M^n, g(t))_{t\in[0,T)}$ $(n \geq 2)$ if and only if its norm of the Riemannian curvature tensor is unbounded as $t \to T$. Therefore a uniform bound for the norm of the Riemannian curvature tensor on $M \times [0, T)$ is enough to extend the Ricci flow over T. Sesum proved that a uniform bound for the norm of Ricci curvature is enough to extend Ricci flow over T. On the other hand, Wang found out some sufficient conditions to extend Ricci flow over T which consist of integral bounds rather than point-wise one. Moreover, Matteo generalized Wang's results using mixed integral norms which is parametrized by $(\alpha, \beta) \in (1, \infty)$.

In the talk, we will give new extendable result on the closed four dimensional Ricci flow under the conditions corresponding to $(\alpha, \beta) = (2, +\infty)$ and $(+\infty, 1)$.

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