WORKSHOP

GEOMETRIC ANALYSIS IN GEOMETRY AND TOPOLOGY 2015

Workshop 「Geometric Analysis in Geometry and Topology 2015 (小林治教授（大阪大・理）の 60 歳還暦をお祝いして)」を下記の要領で開催いたしますのでご案内申し上げます.

記

日時：11 月 9 日（月）〜11 月 12 日（木）･･･ 4 日間
場所：東京理科大学（神楽坂），森戸記念館

★11 月 10 日（火）18：00 より，小林教授の還暦のお祝いしてパーティーを行います.
(パーティーに関しては，時期が近づきましたらご連絡させて頂きます.)

講演者：
・**Bernd Ammann**（Regensburg, Germany）
・**Boris Botvinnik**（University of Oregon, USA）
・**Claude LeBrun**（SUNY at Stony Brook, USA）
・**Rafe Mazzeo**（Stanford University, USA）
・**Jimmy Petean**（CIMAT, Mexico）
・**Harish Seshadri** (Bangalore, India)
・**小林治**（大阪大学・理）
・**金井雅彦**（東京大学・数理）
・**小磯深幸**（九州大学・IMI）
・**小谷元子**（東北大学・理）
・**梅原雅顕**（東京工業大学・情報）
・**山田光太郎**（東京工業大学・理工）
・**納谷信**（名古屋大学・多元数理）
・**松尾信一郎**（大阪大学・理）

スケジュール

･･･････	10:00–11:00	11:30–12:30	14:00–15:00	15:30–16:30	18:00〜
Nov. 9	LeBrun	Mazzeo	Kanai	Matsuo	
Nov. 10	LeBrun	Yamada	Kotani	Kobayashi	**Party**
Nov. 11	Botvinnik	Ammann	Seshadri	Koiso	
Nov. 12	Ammann	Petean	Umehara	Nayatani	

組織員：
・小池直之（東京理科大学・理）
・中村　周 (東京大学・数理)
・古田幹雄 (東京大学・数理)
・小林　治 (大阪大学・理)
・松尾信一郎 (大阪大学・理)
・Rafe Mazzeo (Stanford University, Foreign adviser)
・芥川和雄 (東京工業大学・理工)
・高木章子 (担当事務：東京工業大学・理工)

Program

November 9th (Monday)

10:00–11:00
Claude LeBrun
" Einstein Metrics, Weyl Curvature, and Symplectic 4-Manifolds "
Abstract In this lecture, I will discuss existence and uniqueness results for Einstein metrics on smooth compact 4-manifolds that also admit symplectic structures. The Weyl curvature plays a key role in this discussion, and some of the results. I will describe shed new light on questions regarding the Weyl functional that were articulated by Osamu Kobayashi over thirty years ago.

11:30–12:30
Rafe Mazzeo
" TBA "

12:30–14:00 **Lunchtime**

14:00–15:00
金井 雅彦
" The cross ratio and its folks "
Abstract This is a story about cross ratio and its 'folks', such as the Schwarzian derivative, paraKaehler structures or geodesic currents. Applications of those notions to geometry, the theory of dynamical systems and especially to rigidity problems would be emphasized.

15:30–16:30
松尾 信一郎
" Kobayashi's prescribed scalar curvature problem "
Abstract We will discuss Kobayashi's problem of finding metrics with prescribed scalar curvature and unit total volume, and its close connection with his celebrated Yamabe invariant.

November 10th (Tuesday)

10:00–11:00

Claude LeBrun

" Mass in Kähler Geometry "

Abstract In this lecture, I will explain a simple formula, discovered in joint work with H.-J. Hein, for the mass of any asymptotically locally Euclidean (ALE) Kähler manifold. For ALE scalar-flat Kähler manifolds, the mass turns out to be a topological invariant, depending only on the underlying smooth manifold, the first Chern class of the complex structure, and the Kähler class of the metric. When the metric is actually AE (asymptotically Euclidean), this formula not only implies a positive mass theorem for Kähler metrics, but also yields a Penrose-type inequality for the mass.

11:30–12:30

山田 光太郎

" Analytic extensions of spacelike maximal surfaces in Minkowski 3-space to timelike surfaces "

Abstract A certiain class of spacelike maximal surfaces in Lorentz-Minkowski 3-space have analytic extension to timelike surfaces. We explain this phenomena and construct new embedded examples.

12:30–14:00 **Lunchtime**

14:00–15:00

小谷 元子

" Mathematical Challenge to structural understanding of Materials "

Abstract AIMR challenges to establish a basis of predicting properties/functions of materials by mathematics-materials science collaboration. Three target projects " non- equilibrium materials based on mathematical dynamical system ", " Topological functional materials ", " Multi-scale hierarchical materials based on discrete geometric analysis " are set up. I would like to discuss some emerging results in the projects.

15:30–16:30

小林 治

" Conformal length through Laguerre geometry "

Abstract We assign to a regular curve in the Euclidean space another curve in tempered (and not authentic) Laguerre sphere space. By means of this transform we define conformal invariant length of a curve.

18:00 $\sim$ **Dinner (Party)**

November 11th (Wednesday)

10:00–11:00

Boris Botvinnik

" Topology of the space of metrics with positive scalar curvature "

Abstract We use recent results on the moduli spaces of manifolds, relevant index and surgery theory to study the index-difference map from the space $\mathcal{R}iem^+(W^d)$ of psc-metrics to the space $\Omega^{d+1}KO$ representing the real K-theory. In particular, we show that the index map induces nontrivial homomorphism in homotopy groups $\pi_k\mathcal{R}iem^+(W^d) \to \pi_k\Omega^{d+1}KO$ once the target groups $\pi_k\Omega^{d+1}KO = KO_{k+d+1}$ are not trivial. This work is joint with J.Ebert and O. Randall-Williams.

11:30–12:30

Bernd Ammann

" Topology of the space of D-minimal metrics "

Abstract On a compact connected spin manifold the index theorem by Atiyah and Singer gives lower bounds for the dimension of the kernel of the Dirac operator. Metrics for which the lower bound is attained are called D-minimal. It is conjectured that generic metrics are D-minimal and that non-D-minimal metrics exist on any manifold of dimension at least 3. These conjectures go back to Hitchin's article "Harmonic spinors" where first steps in this program were done, and they were clearly conjectured by C. Bär and M. Dahl. That D-minimal metrics are generic was proven by Bär, Dahl, Humbert and myself using Gromov-Lawson type surgery methods and bordism theory. The existence of non-D-minimal metrics is known in many special cases. In current work with Bunke, Pilca and Nowaczyk we prove the existence of non-D-minimal metrics in dimensions at least 7 using recent progress about the topology of the space of metrics with positive scalar curvature. In most dimensions the main idea is to determine non-trivial homotopy groups in the space of D-minimal metrics.

12:30–14:00 **Lunchtime**

14:00–15:00

Harish Seshadri

" Positive isotropic curvature and self-duality "

Abstract A oriented Riemannian 4-manifold is said to have positive isotropic curvature along self-dual 2-planes if $\frac{s}{6} - W_+ > 0$ as an operator on self-dual 2-forms, where s, W_+ denote the scalar curvature and the self-dual parts of Weyl curvature respectively. Denoting this condition by PIC_+, we prove that PIC_+ is Ricci flow invariant and it is the weakest Ricci flow invariant condition, other than positivity of scalar curvature, in dimension 4. We also discuss some topological consequences of the PIC_+ condition and end with some open questions about the possibility of a topological classification of PIC_+ manifolds by Ricci flow with surgery.

15:30–16:30

小磯 深幸

" On bifurcation and local rigidity of triply periodic minimal surfaces in the three-dimensional Euclidean space "

Abstract We study the structure of the space of all triply periodic minimal surfaces (TPMS) in $\mathbb{R}^3$. The local dimension of this space around each "generic" TPMS is determined. We also discuss the existence of "singularities" of this space and its application to find infinitely many new examples of TPMS's. (Joint work with Paolo Piccione and Toshihiro Shoda.)

November 12th (Thursday)

10:00–11:00

Bernd Ammann

" The Yamabe invariant and surgery "

Abstract In this talk I want to give an overview over some old and new results about the (smooth) Yamabe invariant in joint work with M. Dahl and E. Humbert. We will see for example that the Yamabe invariant of a simply connected compact spin manifold of dimension 5 is between 45 and 79. In dimension 6 it is between 49.9 and 96.3. Similar estimates hold for 2-connected compact spin manifolds with vanishing index in higher dimensions.

Let us give some more details. The (conformal) Yamabe constant of a compact riemannian manifold (M, g_0) is defined as

$$Y(M, [g_0]) := \inf \int_M \mathrm{scal}^g \, dv^g$$

where the infimum runs over all metrics g of volume 1 in $[g_0]$. The (smooth) Yamabe invariant of M is then defined as

$$\sigma(M) := \sup Y(M, [g_0])$$

where the supremum runs over all conformal classes $[g_0]$ on M.

These invariants are tightly related to the existence of metrics of constant scalar curvature in a given conformal class. The invariant $\sigma(M)$ is positive iff M carries a metric of positive scalar curvature. Despite of its simple definition, the Yamabe invariant is extremely difficult to calculate, and it is only known for very few manifolds.

We have proven a formula that estimates the behaviour of $\sigma(M)$ under performing surgery at M, namely if N is obtained by surgery of dimension $k \leq n-3$ from M, then

$$\sigma(N) \geq \min\{\sigma(M), \Lambda_{n,k}\},$$

where $\Lambda_{n,k} > 0$ only depends on $n = \dim M$ and k.

The constants $\Lambda_{n,k} > 0$ arise as Yamabe constants of certain limit spaces which are products of rescaled spheres with the standard hyperbolic spaces.

We found an efficient method to control the Yamabe constants of products spaces provided that both factors are of dimension at least 3. This formula yields positive lower bounds for $\Lambda_{n,k}$ in the case $k \notin \{1, n-3\}$, and also in the case $(n,k) = (6,3)$. This provides the application in dimension 6 and higher. We also found a method to compare the conformal Yamabe constant of our model spaces with the conformal Yamabe invariant of spaces like $\mathbb{R}^2 \times S^2$, $\mathbb{R}^3 \times S^2$ and $\mathbb{R}^2 \times S^3$. The Yamabe constants of the latter spaces were recently calculated by Petean and Ruiz. This method is e.g. sufficient to control $\Lambda_{5,1}$ and $\Lambda_{5,2}$ and thus yields the explicit bound mentioned above for 5-manifolds.

11:30–12:30

Jimmy Petean

" Stability of the Yamabe equation on non-compact manifolds "

Abstract We will discuss the stability of solutions of the Yamabe equation on non-compact manifolds. In the case of the Riemannian product of Euclidean space with a closed manifold M of positive constant scalar curvature there is a unique solution F which depends only on the Euclidean variable. The solution F is actually an extremal function for the Gagliardo-Nirenberg inequality. It is believed that it is a minimizer for the Yamabe functional on the product in some cases (for instance when M is Einstein). We will see that there is a dimensional constant L such that F is stable if and only if the first (positive) eigenvalue of the Laplace operator on M is greater than or equal to L. We will discuss how to compute L to see that F is stable if the metric on M is a Yamabe minimizer.

12:30–14:00 **Lunchtime**

14:00–15:00

梅原 雅顕

" Indices of isolated umbilics on surfaces "

Abstract The index of an isolated umbilic on a given regular surface is the index of the curvature line flow of the surface at that point, which takes values in the set of half-integers. Loewner's conjecture asserts that any isolated umbilic on an immersed surface must have index at most 1. Caratheodory's conjecture asserts the existence of at least two umbilics on an immersed sphere in Euclidean 3-space, which follows immediately from Loewner's conjecture. Let f be a C^1-immersion defined on a unit disk D centered at the origin o such that f is smooth except for being not C^2-differentiable at o. Then o is called a C^1-umbilic if the umbilics of f on $D \setminus \{o\}$ do not accumulate at o. At that point o, we can compute the index of the curvature line flow of f. I talk on the existence of C^1-umbilics with arbitrarily high indices, which is a joint work with Naoya Ando and Toshifumi Fujiyama. Our work implies that more than C^1-regularity is required to prove Loewner's conjecture.

15:30–16:30

納谷 信

" Fixed-point property for uniformly Lipschitz affine actions on a Hilbert space"

Abstract I consider a discrete group and its actions on a Hilbert space by uniformly Lipschitz affine maps. The property that any such action has a fixed-point strengthens Kazhdan's property (T), and distinguishes groups with stronger rigidity among groups with property (T). In this talk, I discuss a proof of Gromov's theorem that if a constant $C > 0$ is fixed, then any uniformly C-Lipschitz action of a random group in the Gromov graph model has a fixed point. I also discuss our recent trial to extend the above theorem by admitting affine maps whose Lipschitz constants may have mild growth with respect to the word length on the group. This is a joint work with Hiroyasu Izeki and Takefumi Kondo.