The 4th South Kyushu workshop on algebra

— Complex Ball Quotients and Related Topics —

- Dates: July 22nd - 25th, 2014
- Venue: Kusunoki Kaikan reception hall, Kumamoto University

— Schedule —

- July 22 (Tue)
  - 13:00 – 14:00: Fumiharu Kato (Kumamoto University)
    Fake projective planes
  - 14:20 – 15:20: Gopal Prasad (University of Michigan)
    Fake projective spaces I
  - 15:40 – 16:40: Gopal Prasad (University of Michigan)
    Fake projective spaces II
  - 17:00 – 18:00: Sai Kee Yeung (Purdue University)
    Geometric aspects of fake projective planes

- July 23 (Wed)
  - 09:30 – 10:30: Hisashi Kasuya (Tokyo Institute of Technology)
    Hodge decomposition of twisted cohomology of complex manifolds
  - 11:00 – 12:00: Shin Kikuta (Tokyo Denki University)
    Positivity of canonical or cotangent bundle over Carathéodory measure hyperbolic manifolds
  - 14:00 – 15:00: Donald Cartwright (University of Sydney)
    Enumerating the fake projective planes
  - 15:30 – 16:30: Tim Steger (Università di Sassari)
    Concrete calculations for lattice subgroups in $PU(2,1)$
  - 17:00 – 18:00: Kenji Ueno (ICU)
    Conformal field theory and topological quantum field theory

- July 24 (Thu)
  - 09:30 – 10:30: Toshiyuki Katsura (Hosei University)
    On the supersingular K3 surface in characteristic 5 with Artin invariant 1
  - 11:00 – 12:00: Alina Vdovina (Newcastle University)
    Combinatorial structure of fake algebraic surfaces
  - 14:00 – 15:00: Jonghae Keum (KIAS)
    Derived Categories of Fake Projective Planes
  - 15:30 – 16:30: Bruno Klingler (Université Paris-Diderot)
    Symmetric differentials and ball quotients
  - 17:00 – 18:00: Bertrand Remy (Institut Camille Jordan)
    Automorphisms of Drinfeld half-spaces over a finite field

- July 25 (Fri)
  - 09:30 – 10:30: Masaaki Murakami (Kagoshima University)
    Non-hyperelliptic deformation of a 2-connected genus 3 hyperelliptic fibration
  - 11:00 – 12:00: Daniel Allcock (University of Texas at Austin)
    Geometric generators for braid-like groups

— Titles and Abstracts —

• Daniel Allcock: Geometric generators for braid-like groups

(joint work with Tathagata Basak) One model for the braid group is: a complex vector space, minus some hyperplanes, modulo a group generated by reflections across them. We call any group arising from this construction "braid-like". We are mostly interested in infinite arrangements in (for example) the complex ball, with the ultimate goal of proving the "monstrous proposal". That is: a particular braid-like group (coming from the complex 13-ball), modulo the squares of the braid-like generators, is (almost the same as) the monster finite simple group. This will require finding generators and relations for this braid-like group, and the subject of the talk will be how we found generators. (We don't know the relations.) The method we used can be applied to other hyperplane arrangements, giving a general tool for finding generating sets for braid-like groups.

• Donald Cartwright: Enumerating the fake projective planes

(Joint work with Tim Steger) Prasad and Yeung showed that to any fake projective plane X there is associated a totally real number field k, a totally complex quadratic extension ℓ of k, and a central simple algebra D of degree 3 with center ℓ. The fundamental group Π of X must be contained as a subgroup of a specific finite index in a maximal arithmetic subgroup $\bar{\Gamma}$ of $PU(2,1)$ associated with (k,ℓ,D). They showed that the triples (k,ℓ,D) must belong to a short finite explicit list. In the cases when D is a division algebra, they showed that there was at least one fake projective plane associated to (k,ℓ,D). For five of the (k,ℓ,D) in their list, D is a matrix algebra, and they left open the question of existence of fake projective planes associated to (k,ℓ,D). I shall discuss some of the details of finding generators and relations for the various $\bar{\Gamma}$'s arising. This enabled us to enumerate the possible Π's in each case, and give presentations for each Π. We showed in particular that there are altogether 50 Π's, and so 50 X's, up to homeomorphism. A significant part of the effort involved showing that fake projective planes do not arise in the five matrix algebra cases. However, an interesting surface did arise in one of these settings. Time permitting, I shall talk about some of its properties.

• Hisashi Kasuya: Hodge decomposition of twisted cohomology of complex manifolds

On Kahler manifolds, Simpson showed Hodge decomposition of twisted cohomology by using Higgs bundles. In this talk, we study Hodge decomposition of twisted cohomology on complex manifolds without Kahler structures by using twisted versions of de Rham, Dolbeault and Bott-Chern cohomologies. Computing cohomologies of homogeneous spaces of solvable Lie groups, we give examples of non-Kahler complex manifolds which admit Hodge decomposition of normal cohomology but do not admit Hodge decomposition of twisted cohomology.

• Fumiharu Kato: Fake projective planes

The first half of this talk will be a general survey of the theories concerning fake projective planes, their constructions and known facts on the classification. The second half will be devoted to the joint-work with Daniel Allcock, in which we give a 'new' fake projective plane that arises from 2-adic uniformization with torsion elements.

• Toshiyuki Katsura: On the supersingular K3 surface in characteristic 5 with Artin invariant 1 (jointwork with S. Kondo and I. Shimada)

We examine the arrangement of smooth rational curves on the supersingular K3 surface X in characteristic 5 with Artin invariant 1, and we construct, by using the superspecial abelian surface, six sets of 16 disjoint smooth rational curves on X and show that they make various configulations in a beautiful symmetry, like $(16)_6$ Kummer configuration. We also explain the relation between these rational curves and the lattice theory.

• Jonghae Keum: Derived Categories of Fake Projective Planes

(Abstract to be announced)

• Shin Kikuta: Positivity of canonical or cotangent bundle over Carathéodory measure hyperbolic manifolds

For any complex manifold, an intrinsic measure, called the Carathéodory measure, is defined on it. It models on the Poinacaré volume form on the complex ball, and is required to have the measure decreasing property for holomorphic maps like the Schwarz lemma. Moreover a complex manifold is called Carathéodory measure hyperbolic if the Carathéodory measure has full support. By this definition, it could be expected that a Carathéodory measure hyperbolic manifold is closely related to a manifold whose canonical or cotangent bundle is positive.

In this talk, I will report my several results on a relation between Carathéodory measure hyperbolicity and algebro-geometric positivity of the canonical or cotangent bundle over a compact complex manifold. It would be especially emphasized that these relations could be numerical, that is, they measure how positivity of the canonical or cotangent bundle increases when Carathéodory measure hyperbolicity becomes stronger. If possible, I will also mention my recent try to understand whether the cotangent bundle is big or not on Carathéodory hyperbolic manifolds.

• Bruno Klingler: Symmetric differentials and ball quotients

In this talk I will describe the relation between the existence of symmetric differentials on a smooth complex projective variety X and the representation theory of its (topological) fundamental group. As an application one obtains new rigidity results for certain ball quotients.

• Masaaki Murakami: Non-hyperelliptic deformation of a 2-connected genus 3 hyperelliptic fibration

Deformation of a fibration is useful in studying the moduli spaces of surfaces of general type. In this talk, I shall give a primitive version of a criterion for a genus 3 hyperelliptic fibration to have a deformation to non-hyperelliptic fibrations. The method is by constructing analogue of Catanese-Pignatelli structure theorem of a non-hyperelliptic genus 3 fibration.

• Gopal Prasad: Fake projective spaces

A fake projective plane (fpp) is a smooth projective complex algebraic surface which has same Betti numbers as the complex projective plane. The first example of an fpp was constructed by David Mumford using p-adic uniformization. It was known that there are only finitely many fpp's and their fundamental groups are arithmetic subgroups of PU(2,1). In a joint work with Sai-Kee Yeung, using number theory, Bruhat-Tits theory, and my formula for the volumes of symmetric spaces modulo arithmetic groups, we have classified fpp's in 28 nonempty classes. This classification, together with extensive computer-assisted computations by Steger and Cartwright, have led to a complete determination of all the fpp's: there are exactly 100 of them. This work has also given us an unexpected algebraic surface. In my talk I will describe this work and its generalization to higher dimensions. For example, we have found fake analogues of complex projective 4-space. Our results have interesting consequences for the cohomology of arithmetic groups and automorphic forms.

• Bertrand Remy: Automorphisms of Drinfeld half-spaces over a finite field

We show that the automorphism group of Drinfeld half-space over a finite field is the projective linear group of the underlying vector space. The proof of this result uses analytic geometry in the sense of Berkovich over the finite field equipped with the trivial valuation. We also take into account extensions of the base field.

• Tim Steger: Concrete calculations for lattice subgroups in $PU(2,1)$

The group of holomorphic diffeomorphisms of $B(\mathbf{C}^2)$ is $PU(2,1)$. All fake projective planes are known to be quotients of $B(\mathbf{C}^2)$ by discrete, cocompact, arithmetic subgroups of $PU(2,1)$. In order to construct the list of all fake planes, Cartwright–Steger had to find generators and relations for 25 particular maximal arithmetic subgroups of $PU(2,1)$, as identified by Prasad–Yeung. Suppose one has available a finite list of matrices belonging to one of the maximal arithmetic subgroups. How can one know that they generate the whole group? How can one construct a set of relations which is known to be sufficient?

The methods used to answer these questions were unsophisticated, robust, and fairly general. The talk will review them. Specifically, the following question will be answered: if 0 is the origin of $B(\mathbf{C}^2)$, if $d(\cdot,\cdot)$ is the invariant metric on $B(\mathbf{C}^2)$, if $r > 0$ is a constant, and if we have constructed a list of elements x of the maximal arithmetic subgroup satisfying $d(x(0), 0) < r$, when can we be sure that our list is complete?

• Kenji Ueno: Conformal field theory and topological quantum field theory

With J. E. Andersen we constructed a modular functor from conformal field theory with gauge symmetry. In the present talk I will show that topological quantum field theory associated to the above modular functor is isomorphic to the one defined by Reshtikhin-Turaev when the gauge group is $\mathrm{sl}(N, \mathbf{C})$. Wenzl's representation of the Hecke algebra via GNS construction plays an important role in our proof.

• Alina Vdovina: Combinatorial structure of fake algebraic surfaces

In 1979 D.Mumford constructed a celebrated example of a fake projective plane, but the same paper contains an outline of a much more general construction of "fake" algebraic surfaces using groups acting on buildings. We'll discuss explicit constructions of such groups, old and new and their connections with the algebraic surfaces.

We'll present new series of irreducible finite characteristic lattices recently obtained in a joint work with Jakob Stix and their application to get new surfaces, including a new fake quadric.

• Sai Kee Yeung: Geometric aspects of fake projective planes

The purpose of the talk is to present some geometric aspects of study of fake projective planes. We would explain some backgrounds related to classification of fake projective planes, some applications as well as some more recent work.