Holomorphic isometries with respect to the Bergman metric and related topics

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Abstract   The study of germs holomorphic isometries between Kähler manifolds equipped with real-analytic metrics dated back to the works of Bochner and Calabi. In his seminal work in 1953 on the topic, Calabi established various theorems of analytic continuation of germs of such maps, especially for maps into space forms such as the possibly infinite-dimensional complex projective space $\mathbb{P}^N$ equipped with the Fubini-Study metric.

We will examine the problem of analytic continuation of germs of holomorphic isometries $f : D \to \Omega$ up to normalizing constants between bounded domains with respect to the Bergman metric. We devise a method for analytic continuation which yields at the same time interior extension as well as analytic continuation beyond the boundary under certain conditions on the Bergman kernel. For instance, we prove that the graph of the germ of such a map extends to an affine-algebraic variety whenever the Bergman kernel $K_D(z, w)$ resp. $K_\Omega(\zeta, \xi)$ is rational in $(z, \overline{w})$ resp. in $(\zeta, \overline{\xi})$.

Of particular interest is the case of holomorphic isometries $f : D \to \Omega$ between bounded symmetric domains. From the Gauss equation one deduces that all such maps are totally geodesic when D is irreducible and of rank ≥ 2. On the other hand, in relation to a problem of Clozel-Ullmo in Arithmetic Geometry we have produced examples of nonstandard holomorphic isometries of the unit disk into polydisks. Recently, we have constructed examples of nonstandard holomorphic isometries of higher dimensional complex unit balls B^n, $n \geq 2$ into irreducible bounded symmetric domains of rank ≥ 2. The construction of such examples is related to the geometric theory of varieties of minimal rational tangents on Fano manifolds. We will also examine some relationships between holomorphic isometries and uniruling by minimal disks.