

# 都の西北 代数幾何学シンポジウム

科学研究費補助金 (基盤研究 (S), No.19104001, 代表 桂 利行) により、下記の通り早稲田大学理工学部において代数幾何学シンポジウムを開催致します。皆様のご来聴を心よりお待ちしております。

## 記

会 期: 2010年11月10日(水)–13日(土)

会 場: 早稲田大学理工学部 55S号館 2階第3会議室 (西早稲田キャンパス)

世話人: 楯 元 (早稲田大学; 03-5286-3015; [kaji@waseda.jp](mailto:kaji@waseda.jp))

講演予定:

### 11月10日水曜日

10:00–11:00 金 泳勲 (ソウル大学)

Limits of algebraic curves

11:20–12:20 大川 新之介 (東京大学大学院数理科学研究科)

On global Okounkov bodies of Mori dream spaces

13:50–14:50 池田 岳 (岡山理科大学理学部応用数学科)

$K$ -theoretic Schubert calculus for Lagrangian Grassmannian

15:10–16:10 澤田 宰一 (東北大学大学院理学研究科数学専攻)

Splitting of Frobenius sandwiches

16:30–17:30 那須 弘和 (東京電機大学情報環境学部)

Obstructions to deforming degenerate curves on a scroll

### 11月11日木曜日

10:00–11:00 安保 広達 (アイダホ大学理学部)

On the secant defectivity of Segre-Veronese varieties I

11:20–12:20 権業 善範 (東京大学大学院数理科学研究科)

Minimal model theory of numerical Kodaira dimension zero  
(数値的小平次元0の極小モデル理論)

13:50–14:50 上原 北斗 (首都大学東京)

McKay correspondence and derived categories

15:10–16:10 川谷 康太郎 (大阪大学大学院理学研究科数学専攻)

Stability conditions on K3 surfaces and subgroups of the autoequivalence group

16:30–17:30 大川 領 (京都大学数理解析研究所)

Generators on toric stacks

## 11月12日金曜日

- 10:00–11:00 金 泳勲 (ソウル大学)  
Birational geometry of moduli spaces of curves in projective varieties
- 11:20–12:20 古川 勝久 (早稲田大学大学院基幹理工学研究科)  
Rational curves on hypersurfaces
- 13:50–14:50 山田 紀美子 (岡山理科大学理学部)  
Birational aspects of moduli of stable sheaves on a surface  
— $K$ -flips and beyond them—
- 15:10–16:10 永井 保成 (東京大学大学院数理科学研究科)  
An explicit study of O’Grady’s examples
- 16:30–17:30 岡田 拓三 (京都大学数理解析研究所)  
Birational unboundedness of rationally connected strict Mori fiber spaces

## 11月13日土曜日

- 10:00–11:00 安保 広達 (アイダホ大学理学部)  
On the secant defectivity of Segre-Veronese varieties II
- 11:20–12:20 深澤 知 (山形大学理学部)  
Projective geometry of the Gauss map in positive characteristic

	水曜日	木曜日	金曜日	土曜日
10:00–11:00	金	安保	金	安保
11:20–12:20	大川 (新)	権業	古川	深澤
13:50–14:50	池田	上原	山田	
15:10–16:10	澤田	川谷	永井	
16:30–17:30	那須	大川 (領)	岡田	

### 交通案内:

- JR 山手線, 地下鉄東京メトロ東西線, 西武新宿線; 「高田馬場駅」下車, 徒歩 15 分.
- JR 山手線; 「新大久保駅」下車, 徒歩 12 分.
- 地下鉄東京メトロ副都心線; 「西早稲田駅」下車, 出口 3 (早大理工方面口) がキャンパスに直結.
- 地下鉄大江戸線; 「東新宿駅」下車, 徒歩 15 分.
- 都バス (池 88) 池袋駅東口→渋谷駅東口, (早 77) 新宿駅西口→早稲田, (高 71) 高田馬場駅前→九段下; 「都立障害者センター前」下車, 徒歩 3 分.

## Abstracts:

November 10 (Wed)

### Limits of algebraic curves

Young-Hoon Kiem (Seoul National University)

A smooth algebraic curve  $C$  in a smooth projective variety  $X$  can be thought of as an ideal sheaf  $I_C$ , an equivalence class of maps  $f : C \rightarrow X$ , or a quotient sheaf  $\mathcal{O}_C$  of the structure sheaf  $\mathcal{O}_X$  of  $X$ . For a degenerating family of smooth curves, different perspectives give rise to different limits as well as different compactifications of the space of smooth curves. With elementary examples, I will discuss why finding limits and compactifying the space of curves are issues of interest. Then I will explain several well-known compactifications via Hilbert scheme, Kontsevich's stable map space and Simpson's moduli space of semistable sheaves. All these compactifications play important roles in enumerative geometry and I will discuss some conjectures comparing the curve counting invariants arising from different compactifications.

### On global Okounkov bodies of Mori dream spaces

Shinnosuke Okawa (The University of Tokyo)

Mori dream space is an interesting class of varieties which includes both toric varieties and log Fano varieties. Global Okounkov body of a variety is a closed cone which encodes some information about the asymptotic behavior of line bundles on the variety. I will talk about an approach toward a question on the rational polyhedrality of global Okounkov bodies of Mori dream spaces.

### $K$ -theoretic Schubert calculus for Lagrangian Grassmannian

Takeshi Ikeda (Okayama University of Science)

I will first give a brief review on the intersection theory of Schubert varieties in the Grassmannian. The most fascinating fact is that Schubert classes are represented by Schur functions. This fact, combined with the Littlewood-Richardson rule (LR-rule) for the Schur functions, enables us to describe explicitly the intersection numbers of Schubert varieties. Next I will explain a result due to A. Buch. He proved a  $K$ -theoretic analogue the LR-rule, which describes the product of structure sheaves of Schubert varieties in the  $K$ -ring of coherent sheaves on the Grassmannian. In the last part, I will present a Pieri formula for the  $K$ -ring of Lagrangian Grassmannian. Our formula expresses a "special" Schubert class times arbitrary Schubert class as a linear combination of some Schubert classes, with explicitly described coefficients. This result is a joint work with H. Naruse and Y. Numata.

### Splitting of Frobenius sandwiches

Tadakazu Sawada (Tohoku University)

Let  $X$  be a smooth variety over an algebraically closed field of positive characteristic. A Frobenius sandwich of  $X$  is a normal variety through which the Frobenius morphism of  $X$  factors. We will discuss Frobenius sandwiches locally and globally from the viewpoint of Frobenius splitting and present classifications of the globally  $F$ -regular Frobenius sandwiches of the projective plane and Hirzebruch surfaces. This is a joint work with N. Hara.

## Obstructions to deforming degenerate curves on a scroll

Hirokazu Nasu (Tokyo Denki University)

Let  $V \subset \mathbb{P}^n$  be a projective variety. A curve  $C$  on  $V$  is said to be degenerate if  $C$  is contained in a hyperplane section  $S = \mathbb{P}^{n-1} \cap V$  of  $V$ . We say that  $C$  is stably degenerate if every small deformation  $C'$  of  $C$  in  $V$  is contained in a deformation  $S'$  of  $S$  in  $V$ . In this talk, given a degenerate curve  $C$  on a scroll  $V$  (a projective space bundle over a projective line), I will discuss the problem of determining whether or not  $C$  is stably degenerate.

November 11 (Thu)

## On the secant defectivity of Segre-Veronese varieties I

Hirotschi Abo (The University of Idaho)

In 1995, Alexander and Hirschowitz proved that higher secant varieties of Veronese varieties have the expected dimension (modulo a fully understand list of exceptions). This work completed the Warning problem for polynomial which had remained unsolved for some time. There are corresponding conjecturally complete lists of defective secant varieties for Segre varieties and for Grassmann varieties. But secant varieties of Segre-Veronese varieties are however less understood. As far as we know, there is no general conjecture on defective secant varieties of such varieties known yet.

The main goal of the talks is to discuss two different inductive approaches to study secant varieties of Segre-Veronese varieties and provide a conjecturally complete list of secant varieties of two-factor Segre-Veronese varieties. If time permits, I would like to discuss new examples of defective secant varieties of three or more factor cases.

## Minimal model theory of numerical Kodaira dimension zero

Yoshinori Gongyo (The University of Tokyo)

In this talk, we prove the existence of minimal models of numerical Kodaira dimension 0 after we review a log minimal model program with scaling, works of Birkar-Cascini-Hacon-McKernan and a numerical Zariski decomposition after Nakayama. Moreover we also give new proof of the abundance theorem for log canonical pairs in the case of numerical Kodaira dimension 0 proved by Kawamata.

## McKay correspondence and derived categories

Hokuto Uehara (Tokyo Metropolitan University)

The derived equivalence between an algebraic variety and some algebra is the culmination of the McKay correspondence. We explain when such equivalences exist, some examples and applications of it.

## Stability conditions on K3 surfaces and subgroups of the autoequivalence group

Kotaro Kawatani (Osaka University)

Let  $X$  be a projective K3 surface and  $D(X)$  the bounded derived category of  $X$ . Then the space of stability conditions  $Stab(X)$  on  $D(X)$  is non empty by virtue of Bridgeland. In this talk we show that the special locus  $U(X)$  in  $Stab(X)$  strongly depends on the K3 surface  $X$  rather than the category  $D(X)$ .

## Generators on toric stacks

Ryo Ookawa (RIMS, Kyoto University)

Bondal conjectures an existence of a generator for any smooth complete toric variety by using Frobenius push-forward. We introduce an extended concept, toric stacks and hope such an existence for any complete toric stack. We show some examples of toric Fano stacks which have generators by Frobenius push-forward.

November 12 (Fri)

## Birational geometry of moduli spaces of curves in projective varieties

Young-Hoon Kiem (Seoul National University)

I will briefly explain the construction of Hilbert schemes and the definitions of (semi)stable sheaves as well as Simpson's construction by geometric invariant theory. Then I will show how elementary modification of sheaves may enable us to compare the compactifications introduced in the first lecture. Precise comparison results will be provided for genus 0, degree at most 3 and projective homogeneous  $X$  (Joint work with K. Chung). If time permits, I will discuss log MMP for the moduli spaces and some related problems.

## Rational curves on hypersurfaces

Katsuhisa Furukawa (Waseda University)

We study the family of smooth rational curves of degree  $e$  lying on a hypersurface of degree  $d$  in  $\mathbb{P}^n$ . When  $e = 1$ , the smoothness, the dimension and the connectedness of the family have been investigated by W. Barth and A. Van de Ven over  $\mathbb{C}$  and by J. Kollár over an algebraically closed field of arbitrary characteristic. For  $e > 1$ , J. Harris, M. Roth, and J. Starr studied irreducibility, smoothness and the dimension of the family over  $\mathbb{C}$  in the case of  $d < (n + 1)/2$ . In this talk, we will study the family in arbitrary characteristic under the assumption  $e = 2, 3$  and  $d > 1$ , or  $e > 3$  and  $d > 2e - 4$ . In particular, we will discuss a matrix algebra on the decomposition of the normal bundle of a rational curve.

## Birational aspects of moduli of stable sheaves on a surface

— $K$ -flips and beyond them—

Kimiko Yamada (Okayama University of Science)

Let  $H$  be an ample line bundle on a non-singular complex projective surface  $X$ , and  $M(H)$  the coarse moduli scheme of rank-two  $H$ -semistable sheaves with fixed Chern classes on  $X$ . We shall explain the facts that (i) the wall-crossing method on  $M(H)$  induces a natural sequence of  $K$ -flips (flips in the sense of Mori) consisting of moduli schemes, and (ii) when  $X$  is minimal and has positive Kodaira dimension, this sequence of  $K$ -flips terminates in  $M(H_X)$ , where  $H_X$  is an ample line bundle lying closely to  $K_X$ . Moreover, the canonical divisor of  $M(H_X)$  is nef. These facts can be observed as a moduli-theoretic analogue of the minimal model program of  $M(H)$ . We shall also mention what we want to know in order to proceed this moduli-theoretic analogue of MMP.

## An explicit study of O'Grady's examples

Yasunari Nagai (The University of Tokyo)

O'Grady constructed two examples of irreducible symplectic Kähler manifold as a crepant resolution of singular moduli spaces of semistable sheaves on a K3 surface and an Abelian surface. The moduli spaces admits a birational contraction to the corresponding Donaldson-Uhlenbeck compactification. In this talk, we will study the birational geometry of O'Grady's examples over the Donaldson-Uhlenbeck compactifications in an explicit way.

## Birational unboundedness of rationally connected strict Mori fiber spaces

Takuzo Okada (RIMS, Kyoto University)

Birational classification of rationally connected (RC, for short) varieties are reduced to that of RC Mori fiber spaces and the study  $n$ -dimensional Mori fiber spaces are divided into  $n$ -cases depending on the dimension of fibers. In my talk, I will show that, for a given  $n \geq 3$  and  $2 \leq m \leq n - 1$ ,  $n$ -dimensional RC Mori fiber spaces with  $m$ -dimensional fibers are birationally unbounded.

November 13 (Sat)

## On the secant defectivity of Segre-Veronese varieties II

Hirotschi Abo (The University of Idaho)

In 1995, Alexander and Hirschowitz proved that higher secant varieties of Veronese varieties have the expected dimension (modulo a fully understand list of exceptions). This work completed the Warning problem for polynomial which had remained unsolved for some time. There are corresponding conjecturally complete lists of defective secant varieties for Segre varieties and for Grassmann varieties. But secant varieties of Segre-Veronese varieties are however less understood. As far as we know, there is no general conjecture on defective secant varieties of such varieties known yet.

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## Projective geometry of the Gauss map in positive characteristic

Satoru Fukasawa (Yamagata University)

Many pathological phenomena appearing in positive characteristic seem to be caused by the inseparability of the Gauss map. In this talk, the speaker will explain the relation between such phenomena and the inseparability of the Gauss map. Then, there is a natural question: *What projective variety admits a (birational) embedding with inseparable Gauss map?* The speaker will talk about (at least) two results to answer this question. One is that any variety admits a birational embedding with inseparable Gauss map. Another is that some familiar examples of projective varieties do not admit any (biregular) embedding with Gauss map of rank zero. These results are consequences of joint works with K. Furukawa and H. Kaji.