Mark Behrens

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: http://www.nitech.ac.jp/campus/index.htm

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: **Professor Mark Behrens, MIT**

: **Topological automorphic forms**

: I will describe methods relating stable homotopy groups of spheres to the arithmetic of automorphic forms. There is a filtration on the stable stems called the chromatic filtration. The first layer may be detected by K-theory, and the second layer may be detected by the cohomology theory of Topological Modular Forms (TMF). After reviewing how this works, I will describe how the nth layer may be studied using cohomology theories of Topological Automorphic Forms associated to Shimura varieties.

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:　　~　　:　　, **Number theoretic background**
I will survey Hilbert's 12th problem, which asks for an explicit form of class field theory, and explain how it is solved in different contexts using the muliplicative group, moduli of elliptic curves, and Shimura varieties.

:　　~　　:　　, **Homotopy theoretic background**
I will review some aspects of the chromatic filtration, and explain how it is studied using the multiplicative group and K-theory (first layer), and moduli of elliptic curves (second layer).

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:　　~　　:　　, **Classification of abelian varieties in characteristic p**
I will outline the Honda-Tate classification of abelian varieties over finite fields, using the concept of p-divisible group.

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: ~ : , **Tate modules and level structures**
I will describe some theorems regarding Tate modules and how to represent isogenies with the data of a level structure.

: ~ : , **Polarizations**
I will review the theory of polarizations, and classify polarizations of abelian varieties over finite fields.

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: ~ : , **Shimura varieties**
After briefly reviewing the notion of a stack, I will define the Shimura varieties we will be studying.

: ~ : , **Deformation theory**
I will discuss the deformation theory of formal groups, p-divisible groups, and mod p points of our Shimura varieties.

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: ~ : , ( ) **Wrapping spheres around spheres**
I will discuss the topological problem of classifying ways to wrap n-dimensional spheres around k-dimensional spheres. I will describe the patterns that emerge in this classification, and relate them to some elementary number theory.

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: ~ : , **Height n locus**
I will describe a zero dimensional subvariety of particular importance to the nth chromatic layer in homotopy theory.

: ~ : , **Topological automorphic forms**
I will describe a generalization of the Hopkins-Miller theorem (due to Jacob Lurie) and explain how the deformation theory allows us to construct the cohomology theory of Topological Automorphic forms (TAF).

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: ~ : , Q-**spectrum**
I will show that the adelic points of a unitary acts on TAF, the homotopy fixed points of this action is a spectrum Q_U. This spectrum Q_U is the main object of interest. I will prove

1. the spectrum admits a resolution given by the action of U on its building
2. the $K(n)$-localization of Q_U is an approximation to the nth chromatic layer in homtopy theory.

: ~ : , **Examples**
I will describe some examples

$n = 1$: Class field theory and K-theory

$n = 2$: Basically reduces to elliptic curve theory and TMF

$n = p - 1$: I will show that maximal finite subgroups of the Morava stabilizer group may be realized.

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