乞掲示

名工大ホモトピー論集会０６－１

文部科学省科学研究費基盤研究 (B)(1) 課題番号 16340015 (代表 南 範彦)

による研究集会を開催致しますのでご案内申し上げます.

日時 ： ２００６年７月５日 (水) ～ ７月７日 (金)
会場 ： 名古屋市昭和区御器所町名古屋工業大学
共通２３号棟 (古墳のすぐ西)・共７講義室 (水曜), 共１０講義室 (木曜)
および, ５２号棟 (教養キャンパス) １０３講義室 (金曜)

・名古屋工業大学ホームページのキャンパス案内: http://www.nitech.ac.jp/campus/index.htm
には、以下の情報へのリンクが張られています。
１ 所在地 （名工大近郊の地図による案内があります。),
２ 交通案内（主な公共交通機関の路線図と名工大までの経路の案内があります。),
３ 建物配置図（名工大敷地内の建物の案内があります。）

講演者: Professor Matthew Ando, University of Illinois at Urbana-Champaign, Urbana IL 61801

講演題名: STRING ORIENTATIONS OF ELLIPTIC COHOMOLOGY

プログラム

７月５日（水）午後: 共通２３号棟 (古墳のすぐ西)・共７講義室
１５:３０ ～ １７:３０ 講演１
７月６日（木）午前: 共通２３号棟 (古墳のすぐ西)・共１０講義室
１０:００ ～ １２:００ 講演２
７月６日（木）午後: 共通２３号棟 (古墳のすぐ西)・共１０講義室
１４:００ ～ １６:００ 講演３, １６:３０ ～ １８:３０ 講演４
７月７日（金） 午前： ５２号棟 (教養キャンパス) １０３講義室
１０:００ ～ １２:００ 講演５

問い合わせ先： 南 範彦 (名古屋工業大学・おもひ領域） nori@nitech.ac.jp

STRING ORIENTATIONS OF ELLIPTIC COHOMOLOGY

MATTHEW ANDO

1. Introduction: The Witten genus, and the string orientation of elliptic cohomology

A *genus* is a ring homomorphism

$$\phi_* : \Omega_* \to R_*,$$

where Ω_* is a bordism ring. The so-called *elliptic genera* are genera taking there values in the rings which arise in the study of elliptic curves, for example modular forms or $\mathbb{Z}[\![q]\!]$.

Witten showed that elliptic genera of a manifold M typically arise as the one-loop amplitude of theory of closed strings moving in M. As an example, he introduced the *Witten genus*

$$w : \Omega_*^{Spin} \to \mathbb{Z}[\![q]\!],$$

and gave a physical proof that if M is a spin manifold with $\frac{p_1}{2}(M) = 0$, then $w(M)$ is a modular form of level 1.

In algebraic topology, genera typically arise as the effect on homotopy rings of an *orientation*

$$\phi : M \to R,$$

where M is a bordism spectrum, and R is a commutative ring spectrum. The ring spectra R which are appropriate for elliptic genera are the so-called elliptic spectra.

It turns out that the Witten genus plays a fundamental role in elliptic cohomology. Hopkins, Rezk, and I have proved that every elliptic spectrum R receives a canonical map

$$MString \to R,$$

naturally in the elliptic spectrum R. Even better, we construct a map

$$MString \to tmf,$$

where tmf is the spectrum of "topological modular forms" of Goerss-Hopkins-Miller.

Some references for this material are [HBJ92, Wit87, Hop95, Hop02, AHS01].

2. Algebraic geometry of even-periodic ring spectra and of the Thom isomorphism

A commutative ring spectrum E is "even periodic" if $\pi_1 E = 0$ and $\pi_2 E$ contains a unit of $\pi_* E$. Any such E is complex-orientable, so $E^0\mathbb{C}P^\infty$ is the ring of functions on a formal group G_E. The splitting principle gives a description of $E^0 X$ in terms of the formal group G_E for many X built from $\mathbb{C}P^\infty$.

I will review this story, with a particular emphasis on the Thom isomorphism and applications to elliptic cohomology. In particular, I will define an elliptic spectrum, and describe the result of Goerss-Hopkins-Miller.

Some references for this material are [AHS01, AHS04, Str99].

Date: June 2006.

3. Units of ring spectra and orientations, with an application to K-theory

If V is a vector bundle over a space X, and if R is a commutative ring spectrum, then $R(X^V)$ is a "twisted form" of $R(X)$. The twist is classified by a map

$$f : X \to BGL_1R,$$

where BGL_1R is the classifying space of the "units" of R. The map f is the obstruction to orientation V in R-theory.

I will review the classical obstruction theory for E_∞ orientations of May-Quinn-Ray [May77], and use it to describe the components of the space of E_∞ maps

$$MSpin \to KO.$$

4. Topological modular forms and its localizations

The construction of the string orientation

$$MString \to tmf$$

proceeds much as in the $MSpin \to KO$ case, but requires some information about the $L_{K(1)}tmf$, where $K(1)$ is Morava K-theory. I will explain how to use the construction of Goerss-Hopkins-Miller to understand $L_{K(1)}tmf$. Some information about congruences for Bernoulli numbers and modular forms will be useful, for which some references are [Ada63, Ser73, Kat73, Kat75, Kob77].

5. The string orientation

I shall show that there is an E_∞ orientation

$$MString \to tmf$$

which refines the Witten genus. Hopkins, Rezk, and I are preparing a paper on this material, but some idea of the argument can be found in [Hop02].

6. The equivariant string orientation and a second construction of the string orientation

Recently, Jacobi Lurie has given a very beautiful and conceptual construction of the string orientation of tmf, using "derived algebraic geometry." I shall describe joint work with John Greenlees, leading to a conceptual construction of the string orientation for rational S^1-equivariant elliptic cohomology. Our work is in some sense a classical analogue of Lurie's.

Lurie has summarized some of his results about elliptic cohomology in a paper available from his web page, `http://www.math.harvard.edu/~lurie/`. The work with Greenlees is in preparation, but the starting point was the papers [And03, Gre05].

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